Adding Close Relations to a Linking Pin Organization with Three Subordinates

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This study proposes a model of adding close relations between a delegate member and every other member of the same level in a complete ternary linking pin organization structure where every pair of nodes which have the same parent in a complete ternary tree is adjacent. We formulate the total shortening distance which is the sum of shortening lengths of shortest paths between every pair of all nodes by adding edges with short lengths and show an optimal additional relation level in numerical examples.

Keywords: Organization Structure, Linking Pin, Complete Ternary Tree, Optimization Modeling

1. Introduction

A linking pin organization has a structure in which relations between members of the same section are added to a pyramid organization structure where there exist only relations between each superior and his direct subordinates. When members and relations between members in the organization correspond to nodes and edges respectively, the linking pin organization structure can be expressed as the structure in which edges between every pair of nodes with the same parent is added to the rooted tree. When the number of subordinates of each member is the number of children of each node, a linking pin organization with K (K = 2, 3, ...) subordinates is a complete K-ary linking pin organization structure where every pair of nodes which have the same parent in a complete K-ary tree is adjacent. The height of the complete K-ary linking pin organization structure expresses the number of levels in the organization.

We have proposed a model of adding relations between two members of the same level and a model of adding relations between every pair of members of the same level in a complete K-ary linking pin organization structure. In these two models we formulated the total shortening distance which is the sum of shortened lengths of shortest paths between every pair of all nodes by adding edges and have obtained the optimal levels of maximizing the total shortening distance (Sawada, 2008). Maximizing the total shortening distance means that the communication of information between every member in the organization becomes the most efficient. Furthermore, for a model of adding relations between a delegate member and every other member of the same level in a complete ternary (K = 3) linking pin organization structure, we formulated the total shortening distance (Sawada, 2021). This model is expressed as all relations have the same length. However, we should consider that adding relations differ from those of complete ternary linking pin organization structure in length. This study proposes a model of adding shorter relations than those of complete ternary linking pin organization structure between a delegate member and every other member of the same level in a complete ternary linking pin organization structure.

2. Formulation of total Shortening Distance

Let $S_H(N)$ denote the total shortening distance, when we add edges between a delegate node and every other node of the same depth N (N = 1, 2, ..., H) in a complete ternary linking pin organization structure of height H (H = 1, 2, ...). The lengths of adding edges are 0.5 while those of edges of the complete ternary linking pin organization structure are 1.

The total shortening distance $S_H(N)$ can be formulated by adding up the following three sums of shortening distances: (i) the sum of shortening distances between every pair of nodes whose depths are equal to or more than N, (ii) the sum of shortening distances between every pair of nodes whose depths are less than N and those whose depths are equal to or more than N and (iii) the sum of shortening distances between every pair of nodes whose depths are less than N in the same way as (Sawada, 2021).

The sum of shortening distances between every pair of nodes whose depths are equal to or more than N is given by

$$A_H(N) = \{M(H-N)\}^2 2 \cdot 3^N \sum_{i=1}^{N-1} i\, 3^i + \{M(H-N)\}^2 0.5(3^N - 1), \tag{1}$$

Where $M(h)$ denotes the number of nodes of a complete ternary tree of height h (h = 0, 1, 2, ...). The sum of shortening distances between every pair of nodes whose depths are less than N and those whose depths are equal to or more than N is given by

$$B_H(N) = 4M(H-N)3^N \sum_{i=1}^{N-2}\sum_{j=1}^{i} j\,3^j + 0.5M(H-N)\{M(N-1)-N\} + M(H-N)\sum_{i=1}^{N-1} i\,3^i, \quad (2)$$

And the sum of shortening distances between every pair of nodes whose depths are less than N is given by

$$C(N) = 2\cdot 3^N \sum_{i=1}^{N-3}\sum_{j=1}^{i} j(i-j+1)3^j + \sum_{i=1}^{N-2} i(N-i-1)3^i, \quad (3)$$

Where we define $\sum_{i=1}^{0}\bullet = 0$, $\sum_{i=1}^{-1}\bullet = 0$ and $\sum_{i=1}^{-2}\bullet = 0$.

From eqs. (1), (2) and (3), the total shortening distance $S_H(N)$ is given by

$$\begin{aligned} S_H(N) &= A_H(N) + B_H(N) + C(N) \\ &= \{M(H-N)\}^2 2\cdot 3^N \sum_{i=1}^{N-1} i\,3^i + \{M(H-N)\}^2 0.5(3^N-1) \\ &+ 4M(H-N)3^N \sum_{i=1}^{N-2}\sum_{j=1}^{i} j\,3^j + 0.5M(H-N)\{M(N-1)-N\} + M(H-N)\sum_{i=1}^{N-1} i\,3^i \\ &+ 2\cdot 3^N \sum_{i=1}^{N-3}\sum_{j=1}^{i} j(i-j+1)3^j + \sum_{i=1}^{N-2} i(N-i-1)3^i. \end{aligned} \quad (4)$$

3. Numerical Examples

Table 1 illustrates the total shortening distance $S_H(N)$ for H = 1, 2, ..., 7 and N = 1, 2, ..., H. Table 1 reveals that $N^*= H$ maximizes $S_H(N)$ for H = 1, 2, ..., 7. These mean that the most efficient level of forming relations between a delegate member and every other member in this model is the lowest level, irrespective of the number of levels in the organization structure.

Table 1 *Total Shortening Distance $S_H(N)$*

N	H = 1	H = 2	H = 3	H = 4	H = 5	H = 6	H = 7
1	1	16	169	1600	14641	132496	1194649
2	-	62	944	9854	92960	849662	7686224
3	-	-	1500	19755	198396	1849203	16835580
4	-	-	-	24970	297118	2902474	26818750
5	-	-	-	-	343361	3819584	36621209
6	-	-	-	-	-	4228020	44956506
7	-	-	-	-	-	-	48587488

4. Conclusions

This study considered obtaining an optimal depth $N^*$of which edges between a delegate node and every other node in a complete ternary linking pin organization structure of height H are added. In this paper the lengths of adding edges are 0.5 while those of edges of the complete ternary linking pin organization structure are 1. The total shortening distance which is the sum of shortened lengths of shortest paths between every pair of all nodes by adding edges was formulated. Furthermore, the total shortening distance of this model was illustrated with numerical examples.

5. References

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